

Date: 11/1/18

Chp: Chp. 3:2 → Differentiability

- Obj:
- Determine left-&right-hand derivatives.
 - Determine if a function is differentiable at a pt.

* A function is differentiable over $[a, b]$ if it has a derivative at every interior pt.

and :

$$\lim_{h \rightarrow 0^+} \frac{f(a+h) - f(a)}{h} \quad \left\{ \quad \lim_{h \rightarrow 0^-} \frac{f(b+h) - f(b)}{h}$$

exist @ the endpts.

Ex. 1 - Show that the function has right & left-hand derivatives @ $x=0$, but no derivative @ 0

$$f(x) = \begin{cases} x^2 & x \leq 0 \\ 2x & x > 0 \end{cases} \quad \begin{matrix} \text{(left)} \\ \text{(right)} \end{matrix}$$

$$f(0+h) = (0+h)^2 = h^2$$

$$f(0) = 0$$

$$\lim_{h \rightarrow 0^-} \frac{h^2 - 0}{h} = \frac{h^2}{h} = h = 0$$

$$f(0+h) = 2(0+h) = 2h$$

$$f(0) = 0$$

$$\lim_{h \rightarrow 0^+} \frac{2h - 0}{h} = 2$$

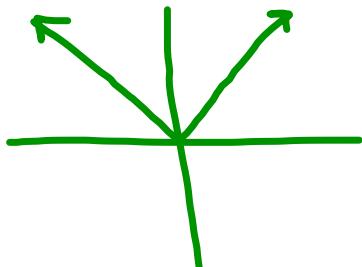
no derivative because $2 \neq 0$!

* A function will have no derivative at a pt $(a, f(a))$ if:

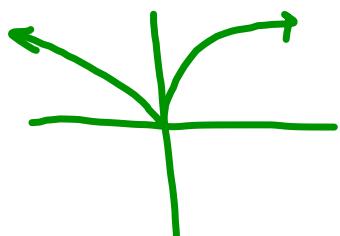
- 1) the left & right-hand limits don't approach the same value.
- 2) $\lim_{x \rightarrow a} \frac{f(x)-f(a)}{x-a}$ fails to exist

Four Ways a $f'(x)$ does not exist:

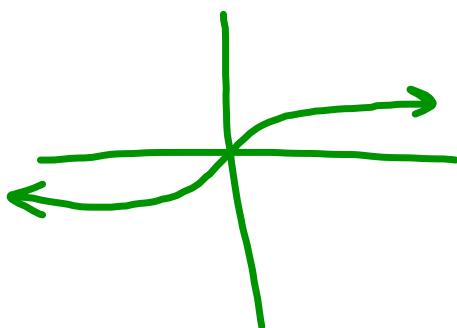
1) Corner $\rightarrow f(x) = |x|$



2) Cusp $\rightarrow f(x) = x^{\frac{2}{3}}$

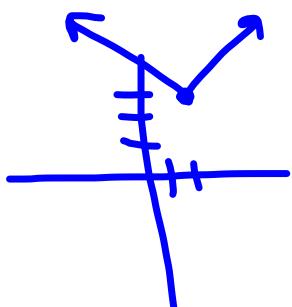


3) Vertical Tangent $\rightarrow f(x) = \sqrt[3]{x}$



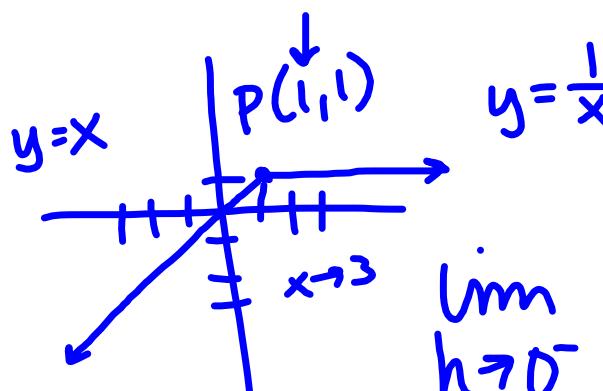
4) Discontinuity \rightarrow V.T., hole, jump

Ex. 2 - Find all the pts where $f(x) = |x-2| + 3$ is not differentiable.



not diff. @ $x=2$

Ex. 3 - Find the right & left-hand derivatives to show that the function is not differentiable @ pt P. Find all of the pts where $f(x)$ is not differentiable.



$$\begin{aligned}f(1+h) &= 1+h \\f(1) &= 1 \\f(1+h) &= \frac{1}{1+h} \\f(1) &= 1\end{aligned}$$

$$\lim_{h \rightarrow 0^-} \frac{1+h-1}{h} = \frac{h}{h} = 1$$

$$\lim_{h \rightarrow 0^+} \frac{\frac{1}{1+h}-1}{h} = \frac{\frac{1-(1+h)}{1+h}}{h}$$

$$\frac{\frac{1-1-h}{1+h}}{h} = \frac{-h}{1+h} \cdot \frac{1}{h} = \frac{-1}{1+h}$$

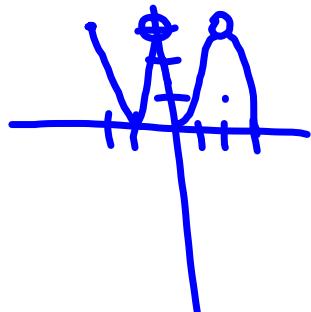
(-1)

Ex.4 - ① closed over the interval
 $D: -2 \leq x \leq 3$

At what domain points does the graph appear to be?

a) differentiable?

$$[-2, -1) \cup (-1, 0) \cup [0, 2) \cup (2, 3]$$



b) continuous but not differentiable?

$$\boxed{x = -1}$$

c) neither, or differentiable?

continuous

$$\boxed{x = 2, 0}$$

Ex.5 - These functions are not differentiable @ $x=0$. Tell whether it is a corner, cusp, vertical tangent, or a discontinuity

a) $y = x^{4/5}$ \rightarrow cusp

b) $y = 3 - \sqrt[3]{x}$ \rightarrow Vert. tang.

c) $y = |\sqrt[3]{x}|$ \rightarrow corner (cusp)

Ex. 6 - Find the derivative using your calculator on $f(x) = x^3 @ x = 2$

$$f'(2) = ?$$

